Session 3
Developing Lift

1.0 Definitions

Acceleration (*a*) - The rate of change of velocity with respect to time, or the change in velocity over a given period of time.

Airfoil - The two-dimensional profile of a wing section.

Coefficient of lift (C_L) - A measure of how efficiently a wing transforms dynamic pressure into a lift force; a proportionality constant which measures how much the pressure changes between the top and bottom of the airfoil.

Density (r) (pronounced "row") - The mass of a substance divided by a unit of volume.

Dynamic pressure (*q*) - The force exerted by a gas in motion.

Momentum (*mV*) - The product of the mass of a substance multiplied by the velocity.

Nautical mile - 6,076 feet. Equal to a one minute arc of latitude at the Earth's equator.

Knot - a measure of speed given as nautical miles per hour.

Static pressure (P_S) - The force exerted by a gas at rest.

Total pressure (P_T) - The sum of the static and dynamic pressures.

Velocity (*V*) - The rate of change of distance with respect to time, or the change in distance over a given period of time.

Wing area - The amount of wing surface an aircraft possesses.

NOTE:

An indepth development of lift is included in sections 7.1 through 7.3 of the text.

2.0 Bernoulli's Equation

Development of the Pressure Relationship

In the mid-1800's, a scientist by the name of Daniel Bernoulli used Newton's second law to mathematically explain the pressure relationship between a moving fluid and a fluid at rest. Bernoulli phrased the relationship as;

"The pressure of a mass of moving fluid in an open area is a constant; and that constant is the sum of the static pressure plus the dynamic pressure."

Static pressure is presented to most people daily in the form of the barometer reading given by the local weather forecaster. This reading is in the form of inches (or millimeters) of mercury which can be directly converted to a pressure, normally, 14.7 lbs/sq. in. at sea level. Changes in static pressure can be felt when our ears feel like they have filled up, causing us to "pop" our ears. Dynamic pressure is perhaps a bit more obscure, but none the less common. When you put your hand out the window of a moving car, you appear to feel the "force of the wind" pushing your hand backward. What you are really experiencing is the dynamic pressure (*q*) which is the result of the velocity of the air mass (or in this case, the velocity of the car through the air mass). The only time dynamic pressure can be measured is when the flow of the air mass is brought to rest upon some type of measuring device. In fact, Bernoulli determined that dynamic pressure can be given the numerical value of:

$$q = 1/2\ (\rho)\ V^2$$

where ρ is the fluid density and *V* is the velocity of the fluid mass.

Then Bernoulli's relationship can be written mathematically as:

$$\text{constant} = P_S + q$$

or

$$\text{constant} = P_S + 1/2\ (\rho)\ V^2$$

where P_S is the static pressure of the fluid.

Since the constant is just the sum of the static and dynamic pressures, it is given the name Total Pressure or P_T. Therefore

$$P_T = P_S + 1/2\ (\rho)\ V^2$$

From this relationship, you can see that if the left side of the equation is to remain constant, when one of the pressures on the right increases, the second pressure on the right must make a corresponding decrease. Consider the following example:

***Example*:** A man is standing still in calm air on the beach holding a barometer which indicates 29.92 inches of mercury. If one inch of mercury equals 0.4912 pounds/square inch of pressure, what is the total pressure the man experiences?

***Solution*:**
1. From Bernoulli's equation

$$P_T = P_S + 1/2\,(\rho)\,V^2$$

we can see

$$P_T = 29.92 \text{ inches of Hg} + 0$$

so

$$P_T = (29.92 \text{ inches of Hg})\ (0.4912 \text{ inches of Hg/Pounds per square inch})$$

$$P_T = 14.7 \text{ pounds/square inch}$$

Caution:

The man is standing still in calm air, therefore there is NO dynamic pressure. As a result, the total pressure is simply equal to the static pressure.

In the above example neither the man nor the air was moving. Therefore since the velocity was zero, there was no dynamic pressure to be considered. The early aviation pioneers built upon Bernoulli's equation and went back to Newton's second law to develop the origins of lift.

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3.0 Lift and the Rate of Change of Momentum

We have written Newton's second law as $F = ma$, however we know that acceleration, a, is the rate of change of velocity, V, with respect to time. In other words, acceleration is the measure of how the velocity changes written as "ΔV" or "dV" over a given period of time written as "Δt" or "dt." We also know that momentum is the product of an object's mass multiplied by its velocity, or (mV).

Incorporating these relationships into the equation for Newton's second law, we can rewrite the law as:

$$F = (mass) \times (\Delta Velocity/\Delta time)$$

or

$$F = m\,(\Delta V/\Delta t)$$

If we look at the change in velocity during any given period of time as being the difference between the beginning velocity (V_0) and the ending velocity (V_f) we can again rewrite Newton's second law as

$$F = \frac{m(V_f - V_0)}{dt}$$

Since we also know mV is momentum, then the Second Law can be termed;

F = Rate of Change of Momentum

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Early aerodynamicists used this theory to predict that if a downward rate of change of momentum could be achieved, the equal and opposite force would be in the upward, or "lifting" direction. The difficulty in putting this theory into practice came from determining how to get a rate of change of momentum in the downward direction. Here is where the use of an airfoil became invaluable for a number of reasons.

Looking at the profile of a wing, (Figure 3.1) we can see the shape looks like an elongated water drop laying on its side. This shape is referred to as an airfoil. Usually the top is curved more than the bottom making the upper surface slightly longer than the bottom. Since air passing over the top and bottom must reach the rear of the wing at the same time, the air passing over the top must not only travel faster, but also changes direction and is deflected downward. This actually results in lift being generated due to a rate of change of vertical momentum and a difference in static pressure between the top and bottom of the wing.

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At this point it is important to explain several terms used by pilots and engineers. Looking at the airfoil in Figure 3.2 will help clarify these terms.

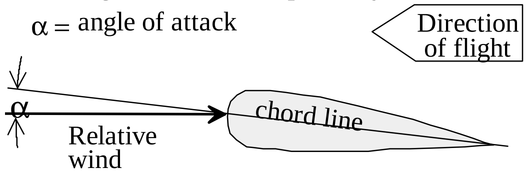

Figure 3.1 Terminology

To begin, the chord line is an imaginary line drawn from the leading edge to the trailing edge of an airfoil. Secondly, the relative wind is the airflow which acts on the airfoil and is parallel to but opposite the direction of flight. The angle between the chord line and the relative wind is called the angle of attack. This is called "alpha" and the symbol used is α. As the angle of attack increases, the change of vertical momentum increases. Additionally, as the angle of attack increases, the coefficient of lift (C_L) increases. The result is an increase in lift. However, there are limits to how much the angle of attack can be increased. Looking at a graph of how the lift coefficient changes with angle of attack, Figure 3.2 shows that at some higher angle of attack, the lift coefficient begins to decrease.

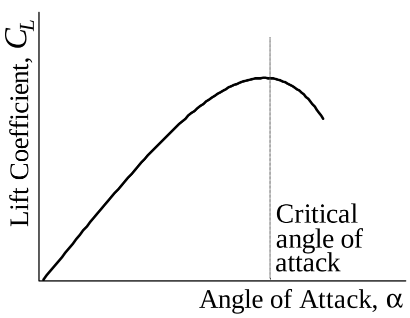

Figure 3.2 Plot of C_L vs α

The angle of attack where the lift coefficient begins to decrease is called the critical angle of attack. Once the critical angle is exceeded, the wing can no longer produce enough lift to support the weight of the aircraft and the wing is said to be "stalled." In other words, the aircraft will stall when the critical angle of attack is exceeded.

To investigate further, first go back to the second law and look at the vertical rate of change of momentum.

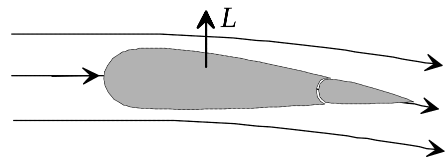

Figure 3.3 Airfoil

Caution:

Recall that momentum is the mass multiplied by the rate of change of velocity in a particular direction. Here we are referring to vertical momentum so we are only concerned with the rate of change of vertical velocity.

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The force, *F*, we are looking for is the lift and is equal to the mass of the air multiplied by the change in vertical velocity of the air over the wing. Whether a wing moves through stationary air, or air is blown over a stationary wing, the physics involved is the same.

Therefore, we can say that in flight there exists an initial velocity of the air in front of the wing (V_0) which has no vertical velocity. In Figure 3.4 we can also see that there is a downward deflection of the air at the rear of the wing (V_f).

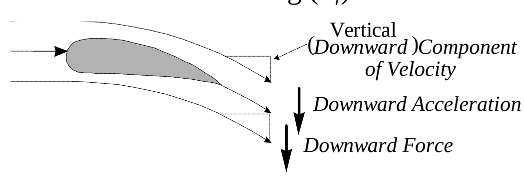


Figure 3.4 Airfoil In Flight

Change in Vertical Momentum

Employing the second law, the rate of change of vertical momentum over the wing the equation becomes:

$$F = \frac{m(V_f - V_0)}{dt}$$

$$F = \frac{m(V_f - 0)}{dt}$$

or

$$F = \frac{m(V_f\ )}{dt}$$

Caution:

Keep in mind these velocities are measured in the vertical direction.

Aerodynamicists knew that density is equal to mass divided by a unit of volume of the air.

$$(\rho) = m/v$$

Taking a unit volume of air then v equals 1, so the equation becomes

$$(\rho) = m/1 = m$$

In this case, the density equals the mass, then the force equaiton can be written.

$$F = \frac{q_{vert}}{Dt}$$

Then according to Newton's third law, the upward force, or lift, would be equal and opposite to the downward rate of change of momentum. Scientists found, however, that it was very difficult to measure the vertical velocity over an airfoil, but measuring the velocity of the air the airfoil was moving through was very simple. This is where they employed Bernoulli's work to help with their research.

Lift Force Opposite Downward
Change of Momentum

4.0 Lift and the Bernoulli Equation

Bernoulli equated the total pressure to the sum of the static and dynamic pressures. The dynamic pressure is a function of the air velocity and the air density.

Density is directly related to temperature, which can be directly measured, and since the air velocity can also be measured, researchers had the dynamic pressure part of Bernoulli's equation well in hand.

$$q = 1/2\ (\rho)\ V^2$$

However, this was only half of the equation. Recall that since the upper surface of the wing is longer, the air must move faster over the top of the wing. Measuring the air velocity would only get the dynamic pressure, not the change in vertical velocity over the wing. Remember the total pressure is the sum of the static and dynamic pressures; and the total pressure must remain constant. So as one increases the other decreases. Then what is needed is how much the static pressure changes over the top of the wing. Since the change in static pressure will be different for different wing shapes, scientists used wind tunnels to measure that static pressure changes between the top and bottom of different wing shapes, assigning each a value referred to as the "Coefficient of Lift" or C_L. Now they had a means of determining all the pressures necessary to find lift. A force, however, is a pressure multiplied by an area.

$$F = (P_T)\,(\,area)$$

Since researchers were dealing with airfoils, and a wing is just several airfoils side-by-side, the logical area to use was the wing area, given the symbol "*S*." At last they had all the ingredients necessary to define lift. The equation is:

$$L = \tfrac{1}{2}\,\rho V^2 S C_L$$

or

$$L = q\,S\,C_L$$

5.0 Summary

We have seen how using Newton's third law, scientists conceived how lift could be developed. Taking that conceptual notion, they employed Bernoulli's pressure relationships to determine how to predict the amount of lift generated by a wing. We know that lift and weight are equal and opposite forces so let's look at one final example to tie all of this together.

6.0 Measures of Performance

1 What is momentum?

2 How does momentum relate to Newton's second law?

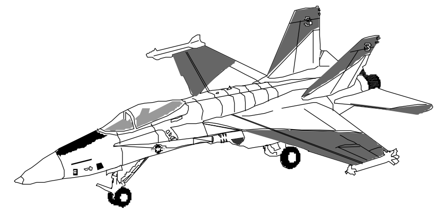
3 What is lift?

7.0 Example

***Problem*:**

To what speed must an aircraft be propelled before it can become airborne given the following information:

Aircraft weight: 26,000 pounds
Wing area: 600 square feet
Air density: 0.002378 slugs/ft3
Lift Coefficient: 0.8



***Solution*:**

1. The lift equation is: $L = \tfrac{1}{2}\,\rho V^2 S C_L$
2. We are asked to find the velocity, therefore we must rearrange the equation and solve for the velocity term:

$$V = \left(\frac{2L}{\rho S C_L}\right)^{\frac{1}{2}}$$

3. We are given the weight and since lift must equal weight we can simply put the weight directly into the equation.
4. Substituting the appropriate values into the equation:

$$V = \left(\frac{2(26,000\,lbs)}{\left(0.002378\,\frac{slugs}{ft^3}\right)(600\,ft^2)(0.8)}\right)^{\frac{1}{2}}$$

NOTE:

Recall that a slug/cubic foot is equivalent to a lb-sec^2/cubic foot.

$$V = \left(\frac{52,000\,lbs}{1.14\,\frac{lbs-\sec}{ft}}\right)^{\frac{1}{2}}$$

$$V = 213.4 \text{ ft/sec}$$

5. Now we usually don't speak of aircraft speeds in feet per second, so to convert to miles per hour, multiply feet per second by 0.6818. Then;

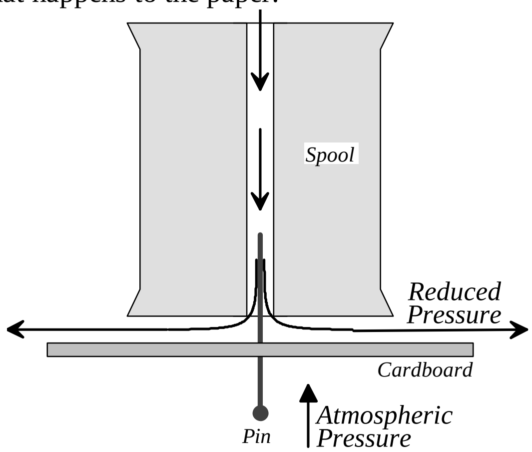
$$V = (213.4\,\tfrac{ft}{\sec})(0.6818\,\tfrac{miles/hour}{ft/\sec})$$

$$V = 145 \text{ mph}$$

Then assuming we keep the wing area, coefficient of lift, and air density constant, we can change the amount of weight we can lift by simply changing the aircraft's velocity. Changing the airplanes velocity requires changing the engine thrust, which is the subject of the next session.

8.0 Suggested Activity

Take an thread spool and hold a piece of cardboard (like from the back of a tablet) with a pin stuck through it in the hole at the bottom of the spool. Holding the spool vertically, blow air through the hole in the top of the spool and watch what happens to the paper.



A jet of air moves horizontally from the hole at the bottom and spreads out over the surface of the cardboard. If air is blown through with sufficient speed, the outward movement of the air at the bottom of the spool will create a low static pressure at the base of the spool. The higher pressure from the atmosphere under the cardboard will hold it close to the spool so you can now let go of the cardboard. This shows the pressure force overcoming the weight of the cardboard.